Exam Kaleidoscope Modern Physics

1 november 2013, 9:00-12:00, 3219.0061

- Answer all questions short and to the point, but complete; write legible.
- Use of a calculator is not allowed.
- $hc = 1240$ eV·nm ; $\hbar c = 200$ eV·nm ; 1 $u = 931.5$ MeV/c^2.
- Put your name and student number on each sheet. Good luck!

1. The wavelengths in the spectra of hydrogen and one-electron ions are well described by the Rydberg formula $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$. Derive this formula and find the value of R. How much energy is needed to remove this single electron of Xe^{53+} ($Z = 54$)? **(2 points)**
   I. The wave-lengths of the light emitted in a hydrogen atom or one-electron ion are determined from the differences in the energy levels of the electron. These are given by $E = -(13.6\,\text{eV})Z^2/n^2$ with Z the charge of the nucleus in units of e and the principle quantum number n an integer > 0. The energy of a photon is determined by the energy difference between the two states involved in a transition, hence $E_\gamma = E_{initial} - E_{final} = -(13.6\,\text{eV})Z^2(1/n^2_{initial} - 1/n^2_{final})$. From the energy-momentum relation we know that $E = \sqrt{(pc)^2 + (mc^2)^2}$. The mass of a photon is zero and thus $pc = E$. The wavelength is related to the momentum via the de Broglie relation, $\lambda = h/p = h \cdot c/E$. From this the Rydberg formula can readily be derived, including the value of $R = 13.6\,\text{eV}\,/\,hc = 10^{-2}\,\text{nm}^{-1}$. **II.** To remove the last electron from an ion (or hydrogen atom), it must be brought from the $n_1 = 1$ state to the $n_2 \to \infty$ state. The amount of energy scales with Z^2 and is 13.6 eV for hydrogen (with $Z = 1$). Thus the amount of energy is given by $E = 13.6 \times 54^3 \simeq 40,000\,\text{eV} = 40\,\text{keV}$.

2. Formulate the uncertainty principle for energy and time. In the reaction $\pi^+ + p \to N^* \to \pi^+ + p$ the unstable particle N^* is formed as a 'resonance' with a "width" $\Delta E = 100$ MeV. Estimate the lifetime of the N^* particle. **(2 points)**
   I. $\Delta E{\cdot}\Delta t \geq \hbar/2$. **II.** As the energy uncertainty is 100 MeV, the corresponding lifetime is obtained as $\tau \simeq \Delta t = \hbar\,/\,2\Delta E$. We know $\hbar c = 200\,\text{MeV{\cdot}fm}$. So $\tau = 200\,\text{MeV} \cdot \text{fm}\,/\,(2 \times 100\,\text{MeV} \times c) = 1\,\text{fm}/c \simeq 3 \cdot 10^{-24}\,\text{s}$.

3. The isotope $^{64}_{29}Cu$ is unusual in that it can decay by γ, β^-, and β^+ emission. What is the resulting nuclide in each case? **(1 point)**
   I. γ-decay: A and Z don't change, hence $^{64}_{29}Cu$ **II.** β^--decay: A doesn't change and Z goes up by one; hence $^{64}_{30}Zn$ **III.** β^+-decay: A doesn't change and Z goes down by one; hence $^{64}_{28}Ni$

4. The alpha-particles from a given alpha-emitting nuclide are generally mono-energetic; that is, they all have the same kinetic energy. But the beta particles from a beta-emitting nuclide have a spectrum of energies. Explain the difference between the two cases. **(1 point)**
   α-decay is a two-body decay. Energy and momentum conservation dictate that the α has a fixed energy. In β-decay three particle exist in the final state: the residual nucleus, the β and a neutrino. This results in a continuous spectrum of β energies.

5. The salt KCl has a density of 1.99 g/cm^3. What is the molecular mass of this molecule? Estimate the distance between nearest neighbor K and Cl ions. **(2 points)**

I. The molecular mass is given by the sum of the masses of the K and Cl atom minus the binding energy. The latter is of the order of eV and may be neglected as the atomic masses are of order A GeV/c^2. The atomic masses can be read from the appendix: $m_K = 39.1\,\text{u}$ and $m_{Cl} = 35.5\,\text{u}$, so the molecular weight is $m_{KCl} = m_K + m_{Cl} \simeq 75\,\text{u}$. **II.** The volume V occupied by a single KCl molecule is obtained by dividing the molecular weight by the density of the salt: $V = 75\,u\,/\,1.99\,\text{g/cm}^3 = 38\ \text{u/g cm}^3$. Note that both u and g are units of mass. Hence u/g is a number. Assuming each molecule occupies a cube with side l, the distance between two atoms is equal to $l = \sqrt[3]{V}$. So $l = \sqrt[3]{38} \times \sqrt[3]{u/g}\,\text{cm}$. The first factor is 3.4, the second $1.2 \cdot 10^{-8}$ (from appendix 1, $1\,\text{u} = 1.66 \cdot 10^{-24}\,\text{g}$). So we find $l \simeq 4 \cdot 10^{-8}\,\text{cm}$.

6. What is the basic difference between fission and fusion? Explain why fusion yields more energy per unit mass than fission. **(1 point)**
   I. Fission: the splitting of a large nucleus into smaller ones. Fusion: the merging of two small nuclei into a single larger one. **II.** The energy release is due to the difference in binding energy per nucleon in the inital and final states. This difference is much larger for light nuclei than for heavy ones.

7. At the Large Hadron Collider at CERN, which has a radius of 4.3 km, protons are injected in the main accelerator with an energy of 450 GeV. It takes 20 minutes to accelerate them to 4 TeV. How far do they travel during this period? What is the energy gain in each revolution? **(2 points)**
   I. The injection energy is much larger than the proton mass. Hence we may assume that they move essentially at the speed of light. In 20 minutes they therefore travel $L = 20\,\text{min} \times 60\,\text{s/min} \times 3 \cdot 10^8\,\text{m/s} = 3.6 \cdot 10^{11}\,\text{m}$. **II.** In each revolution they travel $l = 2\pi\rho = 6.3 \times 4.3 \cdot 10^3\,\text{m} = 2.5 \cdot 10^4\,\text{m}$. So the total number of revolutions is $n = L/l = 3.6 \cdot 10^{11}\,\text{m}/2.5 \cdot 10^4\,\text{m/turn} = 1.5 \cdot 10^7$ turns. In this time the energy increases by 4 TeV - 450 GeV = 3.5 TeV. For each turn this yields $3.5 \cdot 10^{12}\,\text{eV}/1.5 \cdot 10^7\,\text{turns} \simeq 2.3 \cdot 10^5\,\text{eV/turn}$.

8. Using the ideas of quantum chromodynamics, would it be possible to find particles made up of two quarks (and no anti-quarks)? What about two quarks and two anti-quarks? Explain. **(1 point)**
   QCD is based on the assumption that observable particles are white, i.e. consist either of equal amounts of each of the three colors: red, green and blue, or of an equal amount of color and anti-color. As each quark has a single color, a particle composed of two quarks can never be white as one of the three colors will be missing. A combination of two quarks and two anti-quarks is OK, as each anti-quark could have the anti-color of one of the quarks.

9. Put the following systems in the correct order of their first appearance after the Big Bang: W boson, deuterium nucleus, electron, free photon, gold nucleus, helium atom, human, neutron, star, quark, supernova. Motivate your ordering, using *e.g.* temperature or the strenght of the various forces. If you cannot, give a (physics-based) explanation why. **(3 points)**
   I. W boson, electron, quark: these are all elementary particles that can be created immediately after the Big Bang. No binding is required, so these particles can have high energies;
   II. neutron: requires the binding of quarks using the strong force. The temperature needs to be low enough that the kinetic energy is smaller than the binding energy;
   III. deuterium nucleus: requires the binding between a proton and a neutron, which thus first have to be formed. Nuclear binding energies are of order MeV (here it's 2.2 MeV), which has to be below the thermal energy of the protons and neutrons;
   IV. helium atom, free photon: electrons and nuclei are bound into an atom due to the electromagnetic interaction. Typical binding energies are of order eV, with correspondingly lower

temperature. Free photons only exist once the universe is no longer filled with a plasma of charged particle, thus after electrons and nuclei have formed atoms.

V. star: may consist of hydrogen atoms, bound by the gravitational attaction. Binding is exceedingly weak (10^{36} times weaker than the EM interaction), hence star formation can only occur once the temperature of the universe has dropped a lot.

VI. supernova, gold nucleus: supernovae occur at the end of the life-cycle of a star when it has burned up its hydrogen. Gold nuclei cannot be formed in nuclear burning processes and are only formed in the shockwave accompanying a supernova.

VII. human: requires heavy elements formed in supernovae to accumulate on a cold planet such that complicated molecular processes can take place.

10. Fermi problem: Suppose a mountain the size of Mount Everest is found consisting of pure Uranium-235. When used in a nuclear fission reactor, how long (days, weeks, months, ...) could this mountain supply the entire world's population with energy? The density of uranium is $19\,\mathrm{g/cm^3}$. **(3 points)**

    This problem requires the estimation of several quantities: I. the size (volume) of Mount Everest; II. the energy release of U-235 in a nuclear reactor; III. and the energy consumption of the world's population. **I.** We know the heigt of Mount Everest to be of order $h = 10\,\mathrm{km}$ (actually $8848\,\mathrm{m}$). We can assume that it looks like a cone (or a pyramid). The radius r of the circular footprint will be of the same order of magnitude as the height, perhaps twice bigger. Otherwise the mountain would be too sharp or too flat. Let's use $\sqrt{2}$ times, so $14\,\mathrm{km}$. Then the volume is $V = 1/3\pi r^2 h = 2000\,\mathrm{km}^3$. Multiplying with the density gives a total mass of $m = \rho V = 2000 \cdot 10^9\mathrm{m}^3 \times 19 \cdot 10^3\,\mathrm{kg/m}^3 = 4 \cdot 10^{16}\,\mathrm{kg}$. The atomic mass of U-235 is about 235 g/mol. So the mountain would contain about 2×10^{17} moles. After multiplying with Avogadro's constant ($N_A = 6 \cdot 10^{23}/\mathrm{mol}$) the total number of nuclei is obtained as $n \simeq 1 \cdot 10^{41}$ nuclei.

    II. The energy release per fission is about $1\,\mathrm{MeV}$ per nucleon, so about $235\,\mathrm{MeV}$. The total energy supply would then thus be $1 \cdot 10^{41} \times 235\,\mathrm{MeV} = 2.4 \cdot 10^{49}\,\mathrm{eV}$. As $1\,\mathrm{eV} = 1.6 \cdot 10^{-19}\,\mathrm{J}$, this corresponds to $1.5 \cdot 10^{30}\,\mathrm{J}$.

    III. The average energy consuption in the Netherlands is about $2\,\mathrm{kW}$ per household. At about 2 people per household, this comes to about $1\,\mathrm{kW/person}$. Industry and work-related energy consuption might both be similar. The world-wide average will be lower, so let's stick to $1\,\mathrm{kW/person}$. The world population has about 6 billion people, so the world's energy consuption would be $6 \cdot 10^{12}\,\mathrm{W}$. As 1 Watt = 1 Joule per second, the mountain could provide the world's population with energy for $T = 1.5 \cdot 10^{30}\,\mathrm{J} \,/\, 6 \cdot 10^{12}\,\mathrm{W} = 2.5 \cdot 10^{17}\,\mathrm{s}$ or about 8 billion years.

Fundamental Constants

Quantity	Symbol	Approximate Value	Current Best Value†
Speed of light in vacuum	c	3.00×10^8 m/s	2.99792458×10^8 m/s
Gravitational constant	G	6.67×10^{-11} N·m²/kg²	$6.6728(67) \times 10^{-11}$ N·m²/kg²
Avogadro's number	N_A	6.02×10^{23} mol⁻¹	$6.02214179(30) \times 10^{23}$ mol⁻¹
Gas constant	R	8.314 J/mol·K = 1.99 cal/mol·K = 0.0821 L·atm/mol·K	8.314472(15) J/mol·K
Boltzmann's constant	k	1.38×10^{-23} J/K	$1.3806504(24) \times 10^{-23}$ J/K
Charge on electron	e	1.60×10^{-19} C	$1.602176487(40) \times 10^{-19}$ C
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m²·K⁴	$5.670400(40) \times 10^{-8}$ W/m²·K⁴
Permittivity of free space	$\epsilon_0 = (1/c^2\mu_0)$	8.85×10^{-12} C²/N·m²	$8.854187817\ldots \times 10^{-12}$ C²/N·m²
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T·m/A	$1.2566370614\ldots \times 10^{-6}$ T·m/A
Planck's constant	h	6.63×10^{-34} J·s	$6.62606896(33) \times 10^{-34}$ J·s
Electron rest mass	m_e	9.11×10^{-31} kg = 0.000549 u = 0.511 MeV/c^2	$9.10938215(45) \times 10^{-31}$ kg = $5.4857990943(23) \times 10^{-4}$ u
Proton rest mass	m_p	1.6726×10^{-27} kg = 1.00728 u = 938.27 MeV/c^2	$1.672621637(83) \times 10^{-27}$ kg = 1.00727646677(10) u
Neutron rest mass	m_n	1.6749×10^{-27} kg = 1.008665 u = 939.57 MeV/c^2	$1.674927211(84) \times 10^{-27}$ kg = 1.00866491597(43) u
Atomic mass unit (1 u)		1.6605×10^{-27} kg = 931.49 MeV/c^2	$1.660538782(83) \times 10^{-27}$ kg = 931.494028(23) MeV/c^2

† CODATA (3/07), Peter J. Mohr and Barry N. Taylor, National Institute of Standards and Technology. Numbers in parentheses indicate one-standard-deviation experimental uncertainties in final digits. Values without parentheses are exact (i.e., defined quantities).

Other Useful Data

Joule equivalent (1 cal)	4.186 J
Absolute zero (0 K)	−273.15°C
Acceleration due to gravity at Earth's surface (avg.)	9.80 m/s² (= g)
Speed of sound in air (20°C)	343 m/s
Density of air (dry)	1.29 kg/m³
Earth: Mass	5.98×10^{24} kg
Radius (mean)	6.38×10^3 km
Moon: Mass	7.35×10^{22} kg
Radius (mean)	1.74×10^3 km
Sun: Mass	1.99×10^{30} kg
Radius (mean)	6.96×10^5 km
Earth–Sun distance (mean)	149.6×10^6 km
Earth–Moon distance (mean)	384×10^3 km

The Greek Alphabet

Alpha	Α	α	Nu	Ν	ν
Beta	Β	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	Ο	ο
Delta	Δ	δ	Pi	Π	π
Epsilon	Ε	ϵ, ε	Rho	Ρ	ρ
Zeta	Ζ	ζ	Sigma	Σ	σ
Eta	Η	η	Tau	Τ	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	Ι	ι	Phi	Φ	ϕ, φ
Kappa	Κ	κ	Chi	Χ	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	Μ	μ	Omega	Ω	ω

Values of Some Numbers

$\pi = 3.1415927$	$\sqrt{2} = 1.4142136$	$\ln 2 = 0.6931472$	$\log_{10} e = 0.4342945$
$e = 2.7182818$	$\sqrt{3} = 1.7320508$	$\ln 10 = 2.3025851$	1 rad = 57.2957795°

Mathematical Signs and Symbols

$\propto$	is proportional to	$\leq$	is less than or equal to
$=$	is equal to	$\geq$	is greater than or equal to
$\approx$	is approximately equal to	Σ	sum of
$\neq$	is not equal to	$\bar{x}$	average value of x
$>$	is greater than	Δx	change in x
$\gg$	is much greater than	$\Delta x \to 0$	Δx approaches zero
$<$	is less than	$n!$	$n(n-1)(n-2)\ldots(1)$
$\ll$	is much less than		

Properties of Water

Density (4°C)	1.000×10^3 kg/m³
Heat of fusion (0°C)	333 kJ/kg (80 kcal/kg)
Heat of vaporization (100°C)	2260 kJ/kg (539 kcal/kg)
Specific heat (15°C)	4186 J/kg·C° (1.00 kcal/kg·C°)
Index of refraction	1.33

Periodic Table of the Elements§

Symbol — Cl 17 — Atomic Number
Atomic Mass§ — 35.453
3p⁵ → Electron Configuration (outer shells only)

Group I	Group II	Transition Elements										Group III	Group IV	Group V	Group VI	Group VII	Group VIII
H 1 1.00794 $1s^1$																	He 2 4.002602 $1s^2$
Li 3 6.941 $2s^1$	Be 4 9.012182 $2s^2$											B 5 10.811 $2p^1$	C 6 12.0107 $2p^2$	N 7 14.0067 $2p^3$	O 8 15.9994 $2p^4$	F 9 18.9984032 $2p^5$	Ne 10 20.1797 $2p^6$
Na 11 22.98976928 $3s^1$	Mg 12 24.3050 $3s^2$											Al 13 26.9815386 $3p^1$	Si 14 28.0855 $3p^2$	P 15 30.973762 $3p^3$	S 16 32.065 $3p^4$	Cl 17 35.453 $3p^5$	Ar 18 39.948 $3p^6$
K 19 39.0983 $4s^1$	Ca 20 40.078 $4s^2$	Sc 21 44.955912 $3d^14s^2$	Ti 22 47.867 $3d^24s^2$	V 23 50.9415 $3d^34s^2$	Cr 24 51.9961 $3d^54s^1$	Mn 25 54.938045 $3d^54s^2$	Fe 26 55.845 $3d^64s^2$	Co 27 58.933195 $3d^74s^2$	Ni 28 58.6934 $3d^84s^2$	Cu 29 63.546 $3d^{10}4s^1$	Zn 30 65.409 $3d^{10}4s^2$	Ga 31 69.723 $4p^1$	Ge 32 72.64 $4p^2$	As 33 74.92160 $4p^3$	Se 34 78.96 $4p^4$	Br 35 79.904 $4p^5$	Kr 36 83.798 $4p^6$
Rb 37 85.4678 $5s^1$	Sr 38 87.62 $5s^2$	Y 39 88.90585 $4d^15s^2$	Zr 40 91.224 $4d^25s^2$	Nb 41 92.90638 $4d^45s^1$	Mo 42 95.94 $4d^55s^1$	Tc 43 (98) $4d^55s^2$	Ru 44 101.07 $4d^75s^1$	Rh 45 102.90550 $4d^85s^1$	Pd 46 106.42 $4d^{10}5s^0$	Ag 47 107.8682 $4d^{10}5s^1$	Cd 48 112.411 $4d^{10}5s^2$	In 49 114.818 $5p^1$	Sn 50 118.710 $5p^2$	Sb 51 121.760 $5p^3$	Te 52 127.60 $5p^4$	I 53 126.90447 $5p^5$	Xe 54 131.293 $5p^6$
Cs 55 132.9054519 $6s^1$	Ba 56 137.327 $6s^2$	57–71†	Hf 72 178.49 $5d^26s^2$	Ta 73 180.94788 $5d^36s^2$	W 74 183.84 $5d^46s^2$	Re 75 186.207 $5d^56s^2$	Os 76 190.23 $5d^66s^2$	Ir 77 192.217 $5d^76s^2$	Pt 78 195.084 $5d^96s^1$	Au 79 196.966569 $5d^{10}6s^1$	Hg 80 200.59 $5d^{10}6s^2$	Tl 81 204.3833 $6p^1$	Pb 82 207.2 $6p^2$	Bi 83 208.98040 $6p^3$	Po 84 (209) $6p^4$	At 85 (210) $6p^5$	Rn 86 (222) $6p^6$
Fr 87 (223) $7s^1$	Ra 88 (226) $7s^2$	89–103‡	Rf 104 (267) $6d^27s^2$	Db 105 (268) $6d^37s^2$	Sg 106 (271) $6d^47s^2$	Bh 107 (272) $6d^57s^2$	Hs 108 (277) $6d^67s^2$	Mt 109 (276) $6d^77s^2$	Ds 110 (281) $6d^97s^1$	Rg 111 (280) $6d^{10}7s^1$	112 (285) $6d^{10}7s^2$						

†Lanthanide Series	La 57 138.90547 $5d^16s^2$	Ce 58 140.116 $4f^15d^16s^2$	Pr 59 140.90765 $4f^35d^06s^2$	Nd 60 144.242 $4f^45d^06s^2$	Pm 61 (145) $4f^55d^06s^2$	Sm 62 150.36 $4f^65d^06s^2$	Eu 63 151.964 $4f^75d^06s^2$	Gd 64 157.25 $4f^75d^16s^2$	Tb 65 158.92535 $4f^95d^06s^2$	Dy 66 162.500 $4f^{10}5d^06s^2$	Ho 67 164.93032 $4f^{11}5d^06s^2$	Er 68 167.259 $4f^{12}5d^06s^2$	Tm 69 168.93421 $4f^{13}5d^06s^2$	Yb 70 173.04 $4f^{14}5d^06s^2$	Lu 71 174.967 $4f^{14}5d^16s^2$
‡Actinide Series	Ac 89 (227) $6d^17s^2$	Th 90 232.03806 $6d^27s^2$	Pa 91 231.03588 $5f^26d^17s^2$	U 92 238.0289 $5f^36d^17s^2$	Np 93 (237) $5f^46d^17s^2$	Pu 94 (244) $5f^66d^07s^2$	Am 95 (243) $5f^76d^07s^2$	Cm 96 (247) $5f^76d^17s^2$	Bk 97 (247) $5f^96d^07s^2$	Cf 98 (251) $5f^{10}6d^07s^2$	Es 99 (252) $5f^{11}6d^07s^2$	Fm 100 (257) $5f^{12}6d^07s^2$	Md 101 (258) $5f^{13}6d^07s^2$	No 102 (259) $5f^{14}6d^07s^2$	Lr 103 (262) $5f^{14}6d^17s^2$

§ Atomic mass values averaged over isotopes in percentages they occur on Earth's surface. For many unstable elements, mass of the longest-lived known isotope is given in parentheses. 2006 revisions. (See also Appendix F.) Preliminary evidence (unconfirmed) has been reported for elements 113, 114, 115, 116 and 118.